The distortion analysis of pulse propagation along a rectangular waveguide which is partially filled with a dispersive dielectric film

Erkul Başaran, Hüseyin A. Serim, Serkan Aksoy

Gebze Institute of Technology
Department of Electronics Engineering
41400, Çayırova, Gebze, Kocaeli, TURKEY
E-mail: erkul@gyte.edu.tr, haserim@gyte.edu.tr, saksoy@gyte.edu.tr

Abstract
The transient electromagnetic wave propagation in guided structures is an important problem due to its applications such as material characterization, communication, wave propagation in the ionosphere condition and so on. In the classical manner, the solution of this problem leads to modal field expansion in frequency domain and inherently includes the effect of the geometrical dispersion. However, the effect of material dispersion is also an important phenomenon in practical applications such as vacuum isolation, waveguide filters, impedance matching and phase shifters. In this work, the propagation of Gaussian modulated pulse is investigated along a rectangular waveguide which is partially filled with a Debye type dispersive film material. The Frequency Dependent Finite Difference Time Domain (FD³TD) method is used to solve pulse propagation problem because the traditional Finite Difference Time Domain (FDTD) method is not suitable for the modelling of the dispersive medium. Both the material dispersion and the geometrical dispersion problem are solved in time domain, numerically. The geometrical dispersion effect to the pulse distortion along the rectangular waveguide is compared with the geometrical and the material dispersion effect, which exist together.

1. Introduction
The transient electromagnetic wave propagation along waveguide structures is an important topic due to its wide applications such as modern information and communication technologies, microwave measurement, microwave characterization, data transmission over waveguide structures (like ionosphere conditions) and so on. Especially, wave front behaviour and pulse broadening are important phenomena in the sense of the loss of transmission information. In the classical manner, the solution of this problem leads to modal field expansion in frequency domain and inherently includes geometrical dispersion.

The geometrical dispersion means that every modal field in the waveguide having different cut-off frequency will propagate with different phase velocities. These reality gains much importance especially for wide band signal communications along different shaped waveguide structures.

In practice, wide band signals lead to pulses which can be modulated or unmodulated kinds. In the classical manner, if one thinks that the pulses can be constructed from different Fourier components, the propagation problem of the pulses along the waveguide can be considered as a Fourier domain problem because every Fourier components correspond to a modal field having different phase velocities. This means that the geometrical dispersion will cause the distortion of the pulses which are propagating along the waveguide. This plays an important role because the distortion of the pulses means the loss of information in practical electromagnetic wave transmission systems.

In this work, the Gaussian modulated sinusoidal pulse propagation problem along the rectangular waveguide is numerically solved by using Frequency Dependent Finite Difference Time Domain (FD³TD) method in three dimensional (3D) spaces. First, the problem is focused to the distortion of the pulses along the rectangular waveguide. Thus, the geometrical dispersion effect in the time domain is observed that how to Gaussian modulated pulses are distorted and degraded.

As a second case, a dispersive film is located in the middle of the rectangular waveguide as a kind of obstacle. The dielectric film is modelled as a Debye type dispersive material in the frequency domain. The dispersive character of the film shows an extra dispersion known as a material dispersion. The time domain representation of the dielectric permittivity for the dispersive Debye dielectric material needs the time domain convolution process in the displacement vector constitutive relation.
The time domain convolution process is evaluated by Recursive Convolution (RC) approach based on the FD²TD method. The Anisotropic Perfectly Matched Layer (A-PML) is applied to terminate the computational domain in the front and back sides of the rectangular waveguide section.

2. The Frequency Dependent Finite Difference Time Domain (FD²TD) Method

The Frequency Dependent Finite Difference Time Domain (FD²TD) method is based on the inverse Fourier transformation of the constitutive relation $\tilde{D}(\vec{r}, \omega) = \varepsilon(\omega) \tilde{E}(\vec{r}, \omega)$ for dispersive materials which can be written as

$$ \tilde{D}(\vec{r}, t) = \varepsilon_0 \varepsilon \tilde{E}(\vec{r}, t) + \varepsilon_0 \int_0^t \tilde{E}(\vec{r}, t-\tau) \chi(\tau) d\tau $$  \hspace{1cm} (1.1)$$

where $\varepsilon_0$ is the free space permittivity, $\varepsilon_\infty$ is the material permittivity at infinite frequency, $\chi(\tau)$ is the time domain electric susceptibility function. The Eq. (1.1) can be written in the sense of Yee’s notation $\tilde{D}(\vec{r}, n\Delta t) = \tilde{D}^n$ as

$$ \tilde{D}^n = \varepsilon_0 \varepsilon_\infty \tilde{E}^n + \varepsilon_0 \sum_{m=0}^{n-1} \tilde{E}^{n-m} \int_{m\Delta t}^{(m+1)\Delta t} \chi(\tau) d\tau $$  \hspace{1cm} (1.2)$$

And then using the Maxwell’s equations, the electric field update equation becomes

$$ E_{x_{k+\frac{1}{2},j,k}}^{n+1} = \frac{1}{\varepsilon_\infty + \chi_0} \left[ \varepsilon_\infty E_{x_{k+1/2,j,k}}^n + \varepsilon_0 \sum_{m=0}^{n-1} E_{x_{k+1/2,j,k}}^{n-m} \int_{m\Delta t}^{(m+1)\Delta t} \chi(\tau) d\tau - \int_{(m+1)\Delta t}^{(m+2)\Delta t} \chi(\tau) d\tau \right] $$

$$ + \frac{\Delta t}{\varepsilon_0 (\varepsilon_\infty + \chi_0)} \left[ \begin{array}{c} H_{y_{k+1/2,j+1/2,k}}^{n+1/2} - H_{y_{k+1/2,j-1/2,k}}^{n+1/2} \\ H_{y_{k+1/2,j,k+1/2}}^{n+1/2} - H_{y_{k+1/2,j,k-1/2}}^{n+1/2} \end{array} \right] \Delta y $$

$$ + \frac{\Delta z}{\varepsilon_0 (\varepsilon_\infty + \chi_0)} \left[ \begin{array}{c} H_{z_{k+1/2,j+1/2}}^{n+1/2} - H_{z_{k+1/2,j-1/2}}^{n+1/2} \\ H_{z_{k+1/2,j,k+1/2}}^{n+1/2} - H_{z_{k+1/2,j,k-1/2}}^{n+1/2} \end{array} \right] \Delta z $$

(1.3)

If the dispersive material is a kind of Debye material (like water), the dielectric permittivity function in frequency domain can be written as

$$ \varepsilon(\omega) = \varepsilon_\infty + \frac{\varepsilon_\infty - \varepsilon_s}{1 + j\omega\tau_0} = \varepsilon_\infty + \chi(\omega) $$

(1.4)

where $\varepsilon_\infty$ is the material permittivity at infinite frequency, $\varepsilon_s$ is the material permittivity at zero frequency, $\tau_0$ is the relaxation time of the dispersive material and $\omega$ is angular frequency. If one takes the inverse Fourier transform of the electric susceptibility function $\chi(\omega)$, the time domain electric susceptibility function can be found as follow

$$ \chi(t) = \left( \frac{\varepsilon_s - \varepsilon_\infty}{\tau_0} \right) e^{\frac{t}{\tau_0}} u(t) $$

(1.5)
where \( u(t) \) is the unit step function. Then substituting the time domain susceptibility function into Eq. (1.3), the electric field can be calculated in time domain, numerically. In this sense, the dielectric constant (\textit{real part}) and the dielectric loss (\textit{imaginary part}) can be interpreted as a measure of the ability for a material to store electromagnetic energy and a measure of the ability of a material to dissipate electrical energy into heat, respectively.

3. Numerical Example

A rectangular waveguide is partially filled with a dispersive dielectric film with perfectly conducting walls. The waveguide mode \( TE_{10} \) is excited by a soft electric dipole. The geometry of the problem is shown in Fig. 1.

![Figure 1. The geometry of the problem.](image)

The rectangular waveguide has the dimensions of 6.12 \times 86 \times 4.13 \text{ cm} along the Cartesian Coordinates. In order to satisfy the Courant stability criteria in the FDTD method, the dimension of the unit cell is taken as \( \Delta x = \Delta y = \Delta z = \lambda/20 \). In this case, the computational domain is constructed from 15 \times 200 \times 10 \text{ the unit cells in the Cartesian Coordinates. The number of the FDTD iteration is 1000. As an example, the dispersive film is located in the middle of the rectangular waveguide along \( y \) direction and its thickness is taken as \( \lambda_{\text{guided}}/2 \). The material parameters of the dispersive film are chosen as \( e_r=1.8, e_s=81, \tau_0=9.4 \times 10^{-12} \text{ second} \). These parameters correspond to the material parameter of the water. The electric dipole is located at \( x_s=6.12/2 \text{ cm}, y_s=1 \text{ cm} \) and it has the finite length of the 4 \text{ cm}. Two different observation points are considered. First one is located before the dispersive film at the coordinates of \( (6.12/2) \times (86/4) \times (4.13/2) \text{ cm} \). The other one is located at the coordinates of \( (6.12/2) \times (3 \times 86/4) \times (4.13/2) \text{ cm} \) behind the dispersive film. Two different sources as a sinusoidal signal and a Gaussian modulated pulse signal are used to investigate the reflection and transmission properties of the dispersive dielectric film. The guided frequency of the waveguide is tuned to 2.45 \text{ GHz} which is used for the heating of the water in conventional microwave ovens.

3.1. The sinusoidal excitation

Firstly the sinusoidal signal is used to excite the \( TE_{10} \) rectangular waveguide mode. The operational (\textit{carrier}) frequency \( \omega \) is tuned to perform that the guided frequency of the waveguide becomes 2.45 \text{ GHz}. The sinusoidal signal is applied for all the FDTD iteration time. In Fig. 2, the time dependencies of the electric field \( z \) component are shown for the observation points in the Cartesian unit cells of \( (9,150,5) \text{ (solid line)} \) and \( (9,160,5) \text{ (dotted line)} \).
3.2. The excitation with the Gaussian modulated pulse

The effect of the dispersive dielectric film material dispersion is investigated for the excitation of the Gaussian modulated pulse signal which is used to excite the $\text{TE}_{10}$ rectangular waveguide mode. The Gaussian modulated sinusoidal signal can be formulated as

$$x(t) = e^{-\frac{1}{2}(t-T_{\text{delay}})^2} \sin(\omega(t - T_{\text{delay}}))$$

(1.6)

where $A$ and $T_{\text{delay}}$ are the shape parameters of the time domain signal, $\omega$ is the operational (carrier) frequency. The duration of the signal is chosen as the $1/3$ of the total FDTD time step. The operational frequency $\omega$ is tuned when the guided frequency of the waveguide becomes 2.45 GHz. The distribution of the electric field $z$ component along $x$-$y$ axis ($z=4.13/2 \text{ cm}$) is given in Fig. 3.

Figure 3. The distribution of the electric field $z$ component along $x$-$y$ axis for Gaussian modulated pulse excitation ($z=4.13/2 \text{ cm}$).
In Fig. 4a, the time dependencies of the electric field $z$ component are shown at the observation points $(9,5,5)$ and $(9,15,5)$ as the number of unit cells. In Fig. 4b, the time dependencies of the electric field $z$ component are shown at the observation points $(9,150,5)$ and $(9,160,5)$ as the number of unit cells.

**Figure 4a.** The time dependencies of the electric field $z$ component at the observation points $(9,5,5)$ (solid line) and $(9,15,5)$ (dotted line).

**Figure 4b.** The time dependencies of the electric field $z$ component at the observation points $(9,150,5)$ (solid line) and $(9,160,5)$ (dotted line).

The geometrical dispersion effect of the waveguide is shown in the given results and it is clear that the shape of the transmitted pulse is distorted and damped while the pulse is propagating along the waveguide. In this step, the dispersive dielectric film material dispersion and the geometrical dispersion effects on the Gaussian modulated pulses will be analyzed, together. As an example, the thickness of the dispersive film is chosen as $\frac{\lambda_{\text{guided}}}{2}$. In Fig. 5a and 5b, the comparisons of the time dependencies of the electric field $z$ component are shown at the observation points $(9,15,5)$ and $(9,150,5)$ when the dispersive dielectric film is present and absent.
Figure 5a. The comparison of the time dependencies of the electric field $z$ component at the observation point (9,15,5) when the dispersive film is present (dotted line) and absent (solid line).

Figure 5b. The comparison of the time dependencies of the electric field $z$ component at the observation point (9,150,5) when the dispersive film is present (dotted line) and absent (solid line).

4. Conclusion

The Gaussian modulated pulse propagation problem along a rectangular waveguide is solved using the FD$^2$TD method in time domain, numerically. The effect of the geometrical dispersion is observed that the pulses are distorted and damped when they are propagating along the waveguide. Both the effect of the material and the geometrical dispersion are investigated by filling the waveguide partially with a dispersive dielectric film. The dispersive dielectric film is modelled as a dispersive Debye material. The material parameters of the dispersive film are chosen as similar to the water. The results show how the material dispersion causes extra distortion and degradation for the Gaussian modulated pulse which is propagating along the rectangular waveguide. In future work, the distortion effects of the films having the different thickness and the different material parameters will be analysed in more detail. Especially, the thin film interference phenomena may be investigated in the sense of the destructive and constructive cases. Moreover the pulse distortion effects of the different types of dispersive materials (Lorentz, Cole-Cole etc.) and specific pulse forms having different time dependencies of the excitation sources (step-modulated carrier signal, Gaussian pulse signal, pulse train signal etc.) can be investigated.

References