Types of fracture in metals

- The concept of material strength and fracture has long been studied to overcome failures.
- The introduction of malleable irons during the revolution of material construction led to the perception of brittle and ductile fractures as well as fatigue failure in metals.

Failure in metallic materials can be divided into two main categories:

Ductile failure: Ductile fracture involves a large amount of plastic deformation and can be detected beforehand.

Brittle failure: Brittle fracture is more catastrophic and has been intensively studied.
Ductile vs Brittle Failure

• Classification:

Fracture behavior:

- Very Ductile
- Moderately Ductile
- Brittle

%AR or %EL

- Large
- Moderate
- Small

• Ductile fracture is usually desirable!

Ductile: warning before fracture

Brittle: No warning

Adapted from Fig. 8.1, Callister 7e.
Example: Failure of a Pipe

- **Ductile failure:**
  -- one piece
  -- large deformation

- **Brittle failure:**
  -- many pieces
  -- small deformation

Figures from V.J. Colangelo and F.A. Heiser, *Analysis of Metallurgical Failures* (2nd ed.), Fig. 4.1(a) and (b), p. 66 John Wiley and Sons, Inc., 1987. Used with permission.
## Factors affecting modes of fracture

<table>
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<th>Brittle fracture</th>
<th>Ductile fracture</th>
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<td>Loading condition</td>
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# Ductile vs. Brittle Failure

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<td>Brittle</td>
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</table>

cup-and-cone fracture

brittle fracture
Failure modes

High energy is absorbed by microvoid coalescence during ductile failure (high energy fracture mode) and low energy is absorbed during transgranular cleavage fracture (low energy fracture mode).

Less catastrophic

More catastrophic
Moderately Ductile Failure

- Evolution to failure:
  - necking
  - void nucleation
  - void growth and linkage
  - shearing at surface
  - fracture

- Resulting fracture surfaces (steel)
  - particles serve as void nucleation sites.


Fracture surface of tire cord wire loaded in tension. Courtesy of F. Roehrig, CC Technologies, Dublin, OH. Used with permission.
Microvoid shape

Microvoid shape is strongly influenced by the type of loading.

Uniaxial tensile loading

Equiaxed dimples.

Shear loading

Elongated and parabolic dimples pointing in the opposite directions on matching fracture surfaces.

Tensile tearing

Elongated dimples pointing in the same direction on matching fracture surface.
Theoretical cohesive strength of metals

• In the most basic term, strength is due to the cohesive forces between atoms.
• The attractive and repulsive force acting on the two atoms lead to cohesive force between two atoms which varies in relation to the separation between these atoms, see fig.

The theoretical cohesive strength $\sigma_{\text{max}}$ can be obtained in relation to the sine curve and become.

$$\sigma_{\text{theoretical}} = \sqrt{\frac{E\gamma_s}{a_o}}$$

where

- $\gamma_s$ is the surface energy (J/m$^2$)
- $a_o$ is the unstrained interatomic spacing

Note: Convenient estimates of $\sigma_{\text{max}} \sim E/10$. 

Cohesive force as a function of the separation between atoms.
Example: Determine the cohesive strength of a silica fibre, if \( E = 95 \text{ GPa}, \gamma_s = 1 \text{ J}\cdot\text{m}^{-2}, \) and \( a_o = 0.16 \text{ nm}. \)

\[
\sigma_{\text{max}} = \left( \frac{E\gamma_s}{a_o} \right)^{1/2} = \left( \frac{95 \times 10^9 \times 1}{0.16 \times 10^{-9}} \right)^{1/2} = 24.4 \text{ GPa}
\]

• This theoretical cohesive strength is exceptionally higher than the fracture strength of engineering materials.
• This difference between cohesive and fracture strength is due to inherent flaws or defects in the materials which lower the fracture strength in engineering materials.
• Griffith explained the discrepancy between the fracture strength and theoretical cohesive strength using the concept of energy balance.
Theories of brittle fracture

Griffith theory of brittle fracture

The first analysis on cleavage fracture was initiated by Griffith using the concept of energy balance in order to explain discrepancy between the theoretical cohesive strength and observed fracture strength of ideally brittle material (glass).

Irwin and Orowan modified the Griffith theory to include plastic deformation at the crack tip.
Fractographic observation in brittle fracture

The process of cleavage fracture consists of three steps:

1) Plastic deformation to produce dislocation pile-ups.
2) Crack initiation.
3) Crack propagation to failure.

Distinct characteristics of brittle fracture surfaces:

1) The absence of gross plastic deformation.
2) Grainy or Faceted texture.
3) River marking or stress lines (chevron notches).

Brittle fracture indicating the origin of the crack and crack propagation path
Brittle Failure

Arrows indicate points at which failure originated.
Ideal vs Real Materials

• Stress-strain behavior (Room $T$):

• DaVinci (500 yrs ago!) observed...
  -- the longer the wire, the smaller the load for failure.

• Reasons:
  -- flaws cause premature failure.
  -- Larger samples contain more flaws!

Stress Concentration for A Circular Hole

- Tensile stresses reach 3 times of the applied stress at stress concentration points.
Stress Concentration for An Elliptic Hole

\[ \sigma_y \bigg|_{x=a} = \sigma \left( 1 + \frac{2a}{b} \right) \]

\[ \sigma_{\text{max}} = \sigma \left( 1 + \frac{2a}{b} \right) \]

Radius of curvatour at the tip of the ellipse

\[ \rho = \frac{b^2}{a} \]

\[ \sigma_{\text{max}} = \sigma \left( 1 + 2 \sqrt{\frac{a}{\rho}} \right) \]
Flaws are Stress Concentrators!

\[
\sigma_m = 2\sigma_o \left( \frac{a}{\rho_t} \right)^{1/2} = K_t \sigma_o
\]

where
\[
\begin{align*}
\rho_t &= \text{radius of curvature} \\
\sigma_o &= \text{applied stress} \\
\sigma_m &= \text{stress at crack tip} \\
K_t &= \text{stress concentration factor}
\end{align*}
\]

Adapted from Fig. 8.8(a), *Callister 7e.*
Concentration of Stress at Crack Tip

Adapted from Fig. 8.8(b), *Callister 7e.*
Engineering Fracture Design

• Avoid sharp corners!

\[ K_t = \frac{\sigma_{\text{max}}}{\sigma_o} \]

Adapted from Fig. 8.2W(c), Callister 6e.
(Fig. 8.2W(c) is from G.H. Neugebauer, Prod. Eng. (NY), Vol. 14, pp. 82-87 1943.)
Stress concentrations for different geometrical shapes
Stress Concentration at A Discontinuity

(a)  "Load-flow" lines. (a) No crack. (b) With crack. (c) At crack. (d) Stress distribution for increasing load and plasticity.
Crack Propagation

Cracks propagate due to sharpness of crack tip

- A plastic material deforms at the tip, “blunting” the crack.

Energy balance on the crack

- Elastic strain energy-
  - energy stored in material as it is elastically deformed
  - this energy is released when the crack propagates
  - creation of new surfaces requires energy
Elastic energy released by crack formation:

$$- \frac{\pi \sigma^2 a^2 t}{E}$$

Energy to create new surfaces

$$2(2at) \cdot \gamma_s = 4at \gamma_s$$

$$\Delta U = U - U_0 = -\frac{\pi \sigma^2 a^2 t}{E} + 4at \gamma_s$$

$$\frac{\partial U}{\partial a} = 4t \gamma_s - \frac{2\pi \sigma^2 at}{E} = 0$$

$$\sigma_{cr} = \sqrt{\frac{2E \gamma_s}{\pi a}}$$
When Does a Crack Propagate?

Crack propagates if the applied stress is above critical stress

\[ \sigma_m > \sigma_c \]

\[ \sigma_c = \left( \frac{2E\gamma_s}{\pi a} \right)^{1/2} \]

where

- \( E \) = modulus of elasticity
- \( \gamma_s \) = specific surface energy
- \( a \) = one half length of internal crack

For ductile \( \Rightarrow \) replace \( \gamma_s \) by \( \gamma_s + \gamma_p \)

where \( \gamma_p \) is plastic deformation energy
Griffith theory of brittle fracture

Observed fracture strength is always lower than theoretical cohesive strength.

Griffith explained that the discrepancy is due to the inherent defects in brittle materials leading to stress concentration → lower the fracture strength.

Consider a through thickness crack of length 2a, subjected to a uniform tensile stress $\sigma$, at infinity.

Crack propagation occurs when the released elastic strain energy is at least equal to the energy required to generate new crack surface.

The stress required to create the new crack surface is

$$\sigma = \left( \frac{2E\gamma_s}{\pi a} \right)^{1/2}$$

In plane strain condition, it is given by:

$$\sigma = \left( \frac{2E\gamma_s}{(1-\nu^2)\pi a} \right)^{1/2}$$
Modified Griffith equation

• The Griffith equation is strongly dependent on the crack size $a$, and satisfies only ideally brittle materials like glass.
• Irwin and Orowan suggested Griffith’s equation can be applied to brittle materials undergone plastic deformation before fracture by including the plastic work, $\gamma_p$, into the total elastic surface energy required to extend the crack wall, giving the modified Griffith’s equation as follows

$$\sigma_f = \left[ \frac{2E(\gamma_s + \gamma_p)}{\pi(1-\nu^2)a} \right]^{1/2} \approx \left( \frac{E\gamma_p}{(1-\nu^2)a} \right)^{1/2}, \text{when } \gamma_p \gg \gamma_s$$
Criterion of Failure

\( \gamma_s \) and \( \gamma_p \) are material properties.  \[ G_c = 2(\gamma_s + \gamma_p) \quad (J / m^2) \]

\( G_c \) is called critical energy release rate, and it is a material property.

Applied energy release rate is \( G = \sigma^2 \pi a/E \)

Failure occurs if \( G > G_c \)

In many cases we would like to know the design stress.

For a given crack length, \( a \), failure occurs if \( \sigma > \sigma_{cr} = \sqrt{\frac{G_c E}{\pi a}} \)

Also, if the \( \sigma \) is given we can find the critical crack length for failure.
Linear Elastic Fracture Mechanics

It can be shown that the stress field, $\sigma$, at the tip of a crack is a function of the stress intensity factor, $K$.

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right)$$

Notice: $\sigma \to \infty$ as $r \to 0$

$K$ is a function of the applied stress, the crack length, and the geometry.

$K = f(\sigma, a)$

Usually

$$K = Y\sigma \sqrt{\pi a} \quad (MPa\sqrt{m})$$

Critical $K$ that a material can stand: $K_c \to$ the fracture toughness.

Failure occurs if $K > K_c$
\[ K = Y\sigma \sqrt{\pi a} \]

\( Y = 1 \)

(a)

\[ K = Y\sigma \sqrt{\pi a} \]

\( Y = 1.12 \)

(b)

\[ K = P/t\sqrt{\pi a} \]

(c)

---

Graphs and diagrams showing relationships between variables in a mechanical or material science context.

(d) Graph showing a linear relationship between \( D/d \) and \( Y \).

(e) Graph showing curves for pure bending and 3-point or 4-point bending with different S/W ratios.
G or K, which approach is correct

From Griffith,
\[ \sigma = \sqrt{\frac{GE}{\pi a}} \]

From LEFM,
\[ \sigma = K / \sqrt{\pi a} \]

If we write in terms of material properties
\[ G_c = \frac{K_c^2}{E} \]
Based on data in Table B5, Callister 7e.

Composite reinforcement geometry is: f = fibers; sf = short fibers; w = whiskers; p = particles. Addition data as noted (vol. fraction of reinforcement):

2. (55 vol%) Courtesy J. Corrie, MMC, Inc., Waltham, MA.
4. Courtesy CoorsTek, Golden, CO.
Toughness versus Strength

Fracture toughness in 0.45C–Ni–Cr–Mo steel containing 0.045% S.
Design Against Crack Growth

- Crack growth condition:
  \[ K \geq K_c = Y \sigma \sqrt{\pi a} \]

- Largest, most stressed cracks grow first!

---

**Result 1:** Max. flaw size dictates design stress.

\[
\sigma_{\text{design}} < \frac{K_c}{Y \sqrt{\pi a_{\text{max}}}}
\]

---

**Result 2:** Design stress dictates max. flaw size.

\[
a_{\text{max}} < \frac{1}{\pi} \left( \frac{K_c}{Y \sigma_{\text{design}}} \right)^2
\]
Design Example: Aircraft Wing

- Material has $K_c = 26 \text{ MPa-m}^{0.5}$
- Two designs to consider...

**Design A**
- largest flaw is 9 mm
- failure stress = 112 MPa

**Design B**
- use same material
- largest flaw is 4 mm
- failure stress = ?

- Key point: $Y$ and $K_c$ are the same in both designs.
- Result:

$$
\left( \sigma_c \sqrt{a_{\text{max}}} \right)_A = \left( \sigma_c \sqrt{a_{\text{max}}} \right)_B
$$

- Reducing flaw size pays off!

Answer: $(\sigma_c)_B = 168 \text{ MPa}$
Design against fracture

\[ K = K_c = \sigma \sqrt{\pi a} \]

Material selection | Design stress | Allowable flaw size or NDT flaw detection

\[ \sigma_d = \frac{\sigma_{ys}}{2}, \quad K_{IC} = \sigma_d \sqrt{\pi a} \]

\[ a_c = \frac{1}{\pi} \left( \frac{K_{IC}}{\sigma_{ys}/2} \right)^2 \]

\( a_c \) decreases dramatically with decreasing toughness, especially if the design stress is to be increased.
Loading Rate

- Increased loading rate...
  -- increases $\sigma_y$ and $TS$
  -- decreases %EL

- Why? An increased rate gives less time for dislocations to move past obstacles.
Impact Testing

- Impact loading:
  -- severe testing case
  -- makes material more brittle
  -- decreases toughness

Adapted from Fig. 8.12(b), Callister 7e. (Fig. 8.12(b) is adapted from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, John Wiley and Sons, Inc. (1965) p. 13.)
Temperature

- **Increasing temperature...**
  --increases %EL and $K_C$

- **Ductile-to-Brittle Transition Temperature (DBTT)...**

![Graph showing temperature impact energy relationship]

FCC metals (e.g., Cu, Ni)
BCC metals (e.g., iron at $T < 914°C$) polymers

Brittle ← More Ductile

High strength materials ($\sigma_y > E/150$)

Ductile-to-brittle transition temperature

Adapted from Fig. 8.15, *Callister 7e.*
Temperature vs. Charpy

Charpy energy (J)

Temperature

Steel
Steel (HY-130)
Steel (12 Ni-Maraging)
Steel (18 Ni-Maraging)
Steel (Low alloy Q + T)
Titanium
Aluminum
Steel (4340)
Aluminum

$s_{ys}$
MPa (ksi)
275 (40)
550 (80)
895 (130)
1240 (180)
1380 (200)
825 (120)
760 (110)
260 (38)
1380 (200)
515 (75)
EXTERNAL VARIABLES AFFECTING FRACTURE
Design Strategy: Stay Above The DBTT!

- **Pre-WWII: The Titanic**
  - An oil tanker that fractured in a brittle manner by crack propagation around its girth.

- **WWII: Liberty ships**

- **Problem:** Used a type of steel with a DBTT ~ Room temp.